KNOWLEDGE ORGANISER

NAME & FORM

YEAR 9

AUTUMN TERM

MATHS HIGHER

Year 9 Higher Knowledge Organiser

Maths Knowledge Organiser

THEORETICAL PROBABILITY

Key Concepts

Probabilities can be described using **words** and **numerically**.

We can use **fractions, decimals or percentages** to represent a probability.

Theoretical probability is what should happen if all variables were fair.

All probabilities must **add to 1**.

The probability of something **NOT** happening equals:

$$1 - (probability\ of\ it\ happening)$$

Examples

Probability scale:

Impossible		Even chance		Certain
$\frac{0}{4}$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	$\frac{4}{4}$
0	0.25	0.5	0.75	1
0%	25%	50%	75%	100%

There are only red counters, blue counters, white counters and black counters in a bag.

Colour	Red	Blue	Black	White
No. of counters	9	3	5	2

1) What is the probability that a blue counter is chosen? $\frac{3}{19} = \frac{number\ of\ blue}{total\ number\ of\ counters}$

2) What is the probability that red is **not** chosen? $\frac{10}{19} = \frac{number\ of\ all\ other\ colours}{total\ number\ of\ counters}$

There are only red counters, blue counters, white counters and black counters in a bag.

Colour	Red	Blue	Black	White
No. of counters	9	$3x$	x-5	$2x$

A counter is chosen at random, the probability it is red is $\frac{9}{100}$. Work out the probability is black.

$$9 + 3x + x - 5 + 2x = 100$$
$$6x + 4 = 100$$
$$x = 16$$

Number of black counters = 16 – 5
= 11

Probability of choosing black = $\frac{11}{100}$

Y9 Higher

Key Words

Theoretical
Probability
Fraction
Decimal
Percentage
Certain
Impossible
Even chance

	1	2	3
Prob	5	4	9

1a) Calculate the probability of choosing a 2.
b) Calculate the probability of not choosing a 3.

	1	2	3
Prob	0.37	$2x$	x

2) Calculate the probability of choosing a 2 or a 3.

ANSWERS: 1a) $\frac{4}{18}$ b) $\frac{9}{18}$ 2) P(2) = 0.42 P(3) = 0.21

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RELATIVE FREQUENCY

Key Concepts

Experimental probability differs to theoretical probability in that it is based upon the **outcomes from experiments**. It may not reflect the outcomes we expect.

Experimental probability is also known as the **relative frequency** of an event occurring.

Estimating the number of times an event will occur:

Probability × no. of trials

Examples

Colour	red	blue	white	black
Prob	x	0.2	0.3	x

A spinner is spun, it has four colours on it.
The relative frequencies of each colour are recorded.
The relative frequency of red and black are the same.

a) What is the relative frequency of red?

$$1-(0.2+0.3)=0.5$$

$$x=\frac{0.5}{2}=0.25$$

b) If the spinner is spun 300 times, how many times do you expect it to land on white?

$$0.3\times 300=90$$

Y9 Higher

Key Words

Experimental
Relative frequency
Fraction
Decimal
Probability
Estimate

Number	1	2	3	4
Prob	x	0.46	0.28	x

A spinner is spun which has 1,2,3,4 on it. The probability that a 1 and a 4 are spun are equal.

a) What is the probability that a 4 is landed on?

b) If the spinner is spun 500 times how many times do we expect it to land on a 2?

ANSWERS: a) 0.13 b) 230

Maths Knowledge Organiser

LISTING OUTCOMES AND SAMPLE SPACE

Key Concepts

When there are a number of different possible outcomes in a situation we need a **logical** and **systematic** way in which to view them all.

We can be asked to **list** all possible outcomes e.g. choices from a menu, order in which people finish a race.

We can also use a **sample space diagram**. This records the possible outcomes of two different events happening.

Examples

Starter	Main
Fishcake Melon	Lasagne Beef Salmon

List all of the combinations possible when one starter and one main are chosen.

F, L   M, L
F, B   M, B
F, S   M, S

Note: You can write the initials of each option in a test. You do not need to write out the full word.

Two dice are thrown and the possible outcomes are shown in the sample space diagram below:

	1	2	3	4	5	6
1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
2	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
3	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
4	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
5	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
6	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

1) What is the probability that 2 numbers which are the same are rolled?

$$\frac{6}{36} = \frac{\textit{outcomes where numbers are the same}}{\textit{total number of outcomes}}$$

2) What is the probability that two even numbers are rolled?

$$\frac{9}{36} = \frac{\textit{outcomes where numbers are both even}}{\textit{total number of outcomes}}$$

Y9 Higher

Key Words

List
Outcome
Sample space
Probability

1) Abe, Ben and Carl have a race. List all of the options for the order that the boys can end the race.

		Spinner		
Coin		Red	Green	Blue
	Heads	H,R	H,G	H,B
	Tails	T,R	T,G	T,B

2a) What is the probability that a head is landed on?
b) What is the probability that a head and a green are landed on?

ANSWERS: 1) ABC, ACB, BAC, BCA, CAB, CBA 2a) $\frac{3}{6}$ b) $\frac{1}{6}$

Maths Knowledge Organiser

VENN DIAGRAMS

Key Concepts

Venn diagrams show all possible relationships between different sets of data.

Probabilities can be derived from Venn diagrams. Specific notation is used for this:

$P(A \cap B)$ = Probability of A **and** B

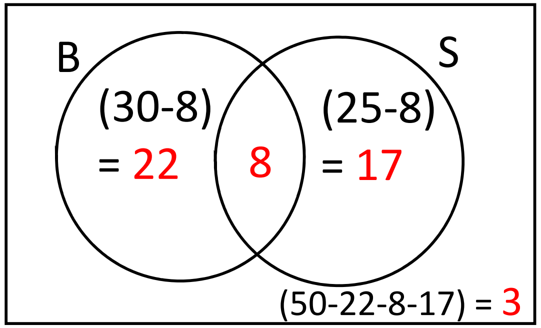
$P(A \cup B)$ = Probability of A **or** B

$P(A')$ = Probability of **not** A

Example

Out of 50 people surveyed:

30 have a brother
25 have a sister
8 have both a brother and sister



a) Complete the Venn diagram

b) Calculate:

i) $P(A \cap B)$ ii) $P(A \cup B)$ iii) $P(B')$

$= \frac{8}{50}$ $= \frac{47}{50}$ $= \frac{20}{50}$

iv) The probability that a person with a sister, does not have a brother.

$= \frac{8}{25}$

40 students were surveyed:

20 have visited France
15 have visited Spain
10 have visited both France and Spain

a) Complete a Venn diagram to represent this information.

b) Calculate:

i) $P(F \cap S)$ ii) $P(F \cup S)$ iii) $P(S')$

iv) The probability someone who has visited France, has not gone to Spain.

ANSWERS: bi) 10/40 ii) 25/40 iii) 25/40 iv) 10/40 = 1/4

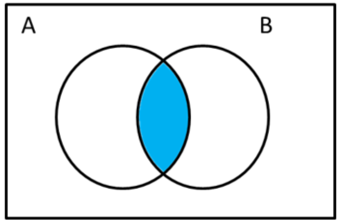
Y9 Higher

Key Words
Venn diagram
Union
Intersection
Probability
Outcomes

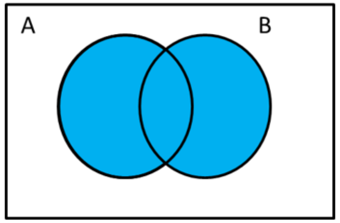
Maths Knowledge Organiser

FURTHER PROBABILITY

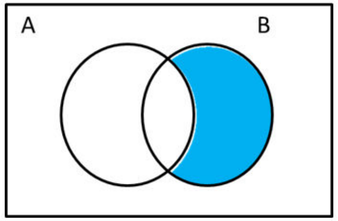
Key Concept



$P(A \cap B)$



$P(A \cup B)$



$P(A' \cap B)$

Y9 Higher

Key Words

Probability: The chance of something happening as a numerical value.

Impossible: The outcome cannot happen.

Certain: The outcome will definitely happen.

Even chance: The are two different outcomes each with the same chance of happening.

Mutually Exclusive: Two events that cannot both occur at the same time.

Formula

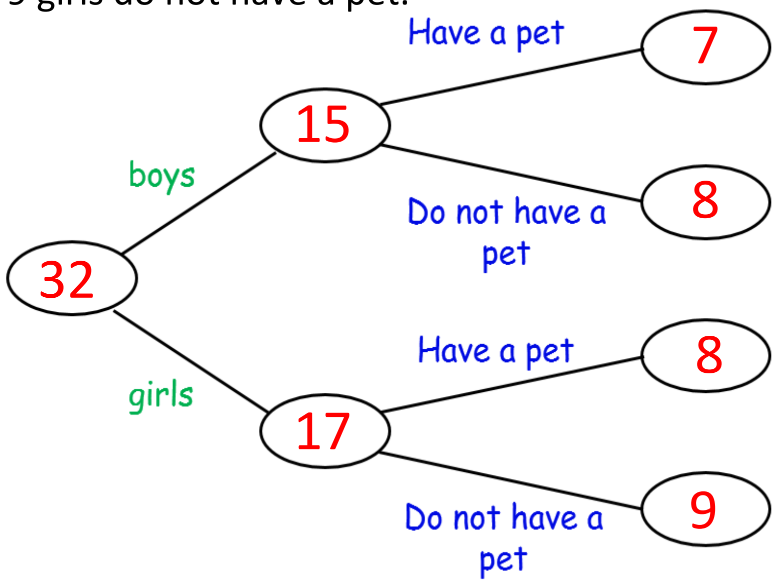
$$P(A \cap B) = P(A) \times P(B)$$

$$P(A \cup B) = P(A) + P(B)$$

$$or \quad (non\ ME) \quad P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Examples

In Hannah's class there are 32 students.
15 of these students are boys.
7 of the boys have a pet.
9 girls do not have a pet.



$$P(boy) = \frac{15}{32}$$

$$P(Girl\ with\ pet) = \frac{8}{32}$$

Questions

1) Draw a two-way table for the question above.
2) Find the probability that a pupil chosen is a boy with no pets.
3) A girl is chosen, what is the probability she has a pet?

ANSWERS: 2) $\frac{8}{32}$ 3) $\frac{8}{17}$

Maths Knowledge Organiser

ORDER OF OPERATIONS

Key Concept

B Brackets

I Indices

D Division

M Multiplication

A Addition

S Subtraction

If a calculation contains the looped calculations work from left to right.

Y9 Higher

Key Words

Operation: In maths these are the functions × ÷ + -.

Commutative: Calculations are commutative if changing the order does not change the result.

Associative: In these calculations you can re-group numbers and you will get the same answer.

Indices: These are the squares, cubes and powers.

Tip

- Put brackets around the calculations which need to be done first.
- Indices also includes roots.

Examples

$$\underbrace{5 \times 4}_{20} - \underbrace{8 \div 2}_{4} = \mathbf{16}$$

$$(2^2 + 6)^2 \times 4 - 8$$

$$(4 + 6)^2 \times 4 - 8$$

$$(10)^2 \times 4 - 8$$

$$100 \times 4 - 8$$

$$400 - 8 = \mathbf{392}$$

Questions

1) $7 - 10 \div 2$
2) $4^3 - 13 \times 4$
3) $21 \div 7 - 2$
4) $12 \div (7 - 3)$
5) $20 \div 2^2$
6) $(16 - 13) \div 3$
7) Place brackets to make the calculation work $20 \div 5 - 3 = 10$

ANSWERS: 1) 2 2) 12 3) 1 4) 3 5) 5 6) 1 7) $20 \div (5 - 3) = 10$

Maths Knowledge Organiser

CALCULATIONS, CHECKING AND ROUNDING

Key Concepts

A value of 5 to 9 rounds the number up.

A value of 0 to 4 keeps the number the same.

Estimation is a result of rounding to one significant figure.

Examples

Round 3.527 to:

a) 1 decimal place

3 . 5 | 2 7 → 3.5

b) 2 decimal places

3 . 5 2 | 7 → 3.53

c) 1 significant figure

3 | . 5 2 7 → 4

Estimate the answer to the following calculation:

$$\frac{46.2 - 9.85}{\sqrt{16.3 + 5.42}}$$

$$\frac{50 - 10}{\sqrt{20 + 5}}$$

$$\frac{40}{5} = 8$$

Y9 Higher

Key Words

Integers
Operation
Negative
Significant figures
Estimate

A) Round the following numbers to the given degree of accuracy

1) 14.1732 (1 d.p.) 2) 0.0568 (2 d.p.) 3) 3418 (1 S.F)

B) Estimate:

1) $\sqrt{4.09 \times 8.96}$

2) $25.76 - \sqrt{4.09 \times 8.96}$

3) $\sqrt[3]{26.64} + \sqrt{80.7}$

4) $\dfrac{\sqrt{6.91 \times 9.23}}{3.95^2 \div 2.023^3}$

ANSWERS A 1) 14.2 2) 0.06 3) 3000 B 1) 6 2) 24 3) 12 4) 4

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INDICES AND ROOTS

Key Concepts

$$a^m \times a^n = a^{m+n}$$

$$a^m \div a^n = a^{m-n}$$

$$(a^m)^n = a^{mn}$$

$$a^{-m} = \frac{1}{a^m}$$

$$a^{\frac{m}{n}} = \sqrt[n]{a^m}$$

$$a^{-\frac{m}{n}} = \frac{1}{\sqrt[n]{a^m}}$$

Examples

Simplify each of the following:

1) $a^6 \times a^4 = a^{6+4}$
$= a^{10}$

2) $a^6 \div a^4 = a^{6-4}$
$= a^2$

3) $(a^6)^4 = a^{6\times 4}$
$= a^{24}$

4) $(3a^4)^3 = 3^3 a^{4\times 3}$
$= 27a^{12}$

5) $a^{-3} = \frac{1}{a^3}$

6) $2a^{-4} = \frac{2}{a^4}$

7) $a^{\frac{1}{2}} = \sqrt[2]{a^1} = \sqrt{a}$

8) $a^{-\frac{1}{2}} = \frac{1}{a^{\frac{1}{2}}} = \frac{1}{\sqrt{a}}$

9) $\left(\frac{25}{16}\right)^{-\frac{1}{2}} = \left(\frac{16}{25}\right)^{\frac{1}{2}}$
$= \sqrt{\frac{16}{25}}$
$= \frac{4}{5}$

Y9 Higher

Key Words

Powers
Roots
Indices
Reciprocal

Write as a single power: 1) $a^3 \times a^2$ 2) $b^4 \times b$ 3) $d^{-5} \times d^{-1}$ 4) $m^6 \div m^2$
5) $n^4 \div n^4$ 6) $\frac{8^4 \times 8^5}{8^6}$ 7) $\frac{4^9 \times 4}{4^3}$

Evaluate : 1) $(3^2)^5$ 2) 2^{-2} 3) $81^{\frac{1}{2}}$ 4) $\left(\frac{1}{9}\right)^{\frac{1}{2}}$ 5) $16^{\frac{3}{2}}$ 6) $27^{-\frac{2}{3}}$

ANSWERS: 1) a^5 2) b^5 3) d^{-6} 4) m^4 5) 1 6) 8^3 7) 4^7
1) 3^{10} 2) $\frac{1}{4}$ 3) 9 4) $\frac{1}{3}$ 5) 64 6) $\frac{1}{9}$

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STANDARD FORM

Key Concepts

We use standard form to write a very large or a very small number in scientific form.

Must be $\times 10$

b is an integer

$$a \times 10^{b}$$

Must be $1 \leq a < 10$

Examples

Write the following in **standard form**:

1) 3000 = 3×10^{3}

2) 4580000 = 4.58×10^{6}

3) 0.0006 = 6×10^{-4}

4) 0.00845 = 8.45×10^{-3}

Calculate the following, write your answer in **standard form**:

1) $(3 \times 10^{3}) \times (5 \times 10^{2})$

$3 \times 5 = 15$
$10^{3} \times 10^{2} = 10^{5}$

15×10^{5}
$= 1.5 \times 10^{6}$

2) $(8 \times 10^{7}) \div (16 \times 10^{3})$

$8 \div 16 = 0.5$
$10^{7} \div 10^{3} = 10^{4}$

0.5×10^{4}
$= 5 \times 10^{3}$

Y9 Higher

Key Words

Standard form

Base 10

Links

Science

A) Write the following in standard form:

1) 74 000 2) 1 042 000 3) 0.009 4) 0.000 001 24

B) Work out:

1) $(5 \times 10^{2}) \times (2 \times 10^{5})$ 2) $(4 \times 10^{3}) \times (3 \times 10^{8})$

3) $(8 \times 10^{6}) \div (2 \times 10^{5})$ 4) $(4.8 \times 10^{2}) \div (3 \times 10^{4})$

ANSWERS: A1) 7.4×10^{4} 2) 1.042×10^{6} 3) 9×10^{-3} 4) 1.24×10^{-6}

B1) 1×10^{8} 2) 1.2×10^{12} 3) 4×10 4) 1.6×10^{-2}

Maths Knowledge Organiser

EXPRESSIONS/EQUATIONS/IDENTITIES AND SUBSTITU

Key Concepts

A **formula** involves two or more letters, where one letter equals an **expression** of other letters.

An **expression** is a sentence in algebra that does NOT have an equals sign.

An **identity** is where one side is the equivalent to the other side.

When **substituting** a number into an expression, replace the letter with the given value.

Examples

1) $5(y + 6) \equiv 5y + 30$ is an identity as when the brackets are expanded we get the answer on the right hand side
2) $5m - 7$ is an expression since there is no equals sign
3) $3x - 6 = 12$ is an equation as it can be solved to give a solution
4) $C = \frac{5(F - 32)}{9}$ is a formula (involves more than one letter and includes an equal sign)
5) Find the value of $3x + 2$ when $x = 5$

   $(3 \times 5) + 2 = 17$

6) Where $A = b^2 + c$, find A when $b = 2$ and $c = 3$

   $A = 2^2 + 3$

   $A = 4 + 3$

   $A = 7$

Y9 Higher

Key Words

Substitute

Equation

Formula

Identity

Expression

Questions

1) **Identify the equation, expression, identity, formula from the list** (a) $v = u + at$ (b) $u^2 - 2as$ (c) $4x(x - 2) = x^2 - 8x$ (d) $5b - 2 = 13$
2) Find the value of $5x - 7$ when $x = 3$
3) Where $A = d^2 + e$, find A when $d = 5$ and $e = 2$

ANSWERS: 1) (a) formula (b) expression (c) identity (d) equation 2) 8 3) $A = 27$

Maths Knowledge Organiser

EQUATIONS IN CONTEXT

Key Concepts

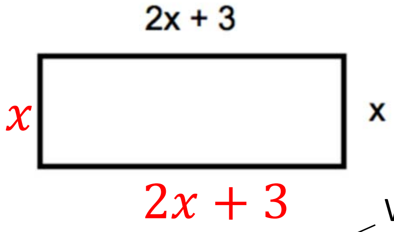
Algebra can be used to support us to find unknowns in a **contextual problem**.

We can always apply a letter to an unknown quantity, to then **set up an equation**.

It will often be used in area and perimeter problems and angle problems in geometry.

Solve to find the value of x when the perimeter is 42cm.

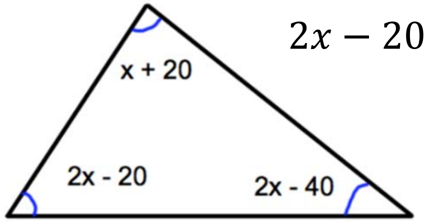
HINT: Write on all of the lengths of the sides.



We know the perimeter is 42cm

$$2x+3+2x+3+x+x=42$$
$$9x+6=42$$
$$6x=36$$
$$x=6$$

Angles in a triangle sum to 180°



$$2x-20+x+20+2x-40=180$$
$$5x-40=180$$
$$5x=220$$
$$x=45$$

Examples

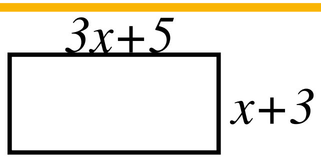
Jane is 4 years older than Tom.
David is twice as old as Jane.
The sum of their ages is 60.
Using algebra, find the age of each person.

Tom = $x \rightarrow 12$
Jane = $x+4 \rightarrow 12+4=16$
David = $2x+8 \rightarrow (2\times 12)+8=32$

$$x+x+4+2x+8=60$$
$$4x+12=60$$
$$4x=48$$
$$x=12$$

Y9 Higher

Key Words
Solve
Term
Inverse operation



1) If the perimeter is 40cm. What is the length of the longest side?

2) Jane is 12 years older than Jack.
Sarah is 3 years younger than Jack.
The sum of their ages is 36.
Using algebra, find the age of each person.

ANSWERS: 1) x = 3 therefore the longest length is 14cm 2) Jack = 9, $Jane = 21$, $Sarah = 6$

Maths Knowledge Organiser

REARRANGE AND SOLVE EQUATIONS

Key Concepts

Solving equations:
Working with inverse operations to find the value of a variable.

Rearranging an equation:
Working with inverse operations to isolate a highlighted variable.

In solving and rearranging we **undo the operations** starting from the last one.

Examples

Solve:

$7p - 5 = 3p + 3$

$-3p \qquad -3p$

$4p - 5 = 3$

$+5 \qquad +5$

$4p = 8$

$\div 2 \qquad \div 2$

$p = 2$

Solve:

$5(x - 3) = 4(x + 2)$

expand $\qquad$ expand

$5x - 15 = 4x + 8$

$-4x \qquad -4x$

$x - 15 = 8$

$+15 \qquad +15$

$x = 23$

Rearrange to make r the subject of the formulae :

$Q = \frac{2r - 7}{3}$

$\times 3 \qquad \times 3$

$3Q = 2r - 7$

$+7 \qquad +7$

$3Q + 7 = 2r$

$\div 2 \qquad \div 2$

$\frac{3Q + 7}{2} = r$

Rearrange to make c the subject of the formulae :

$2(3a - c) = 5c + 1$

expand

$6a - 2c = 5c + 1$

$+2c \qquad +2c$

$6a = 7c + 1$

$-1 \qquad -1$

$6a - 1 = 7c$

$\div 7 \qquad \div 7$

$\frac{6a - 1}{7} = c$

Y9 Higher

Key Words

Solve
Rearrange
Term
Inverse

Links

Science

1) Solve $7(x + 2) = 5(x + 4)$
2) Solve $4(2 - x) = 5(x - 2)$
3) Rearrange to make m the subject $2(2p + m) = 3 - 5m$
4) Rearrange to make x the subject $5(x - 3) = y(4 - 3x)$

ANSWERS: 1) $x = 3$ 2) $x = 2$ 3) $m = \frac{3 - 4p}{7}$ 4) $x = \frac{4y + 15}{5 + 3y}$

Maths Knowledge Organiser

SEQUENCES

Key Concepts

Arithmetic or linear sequences increase or decrease by a common amount each time.

Geometric series has a common multiple between each term.

Quadratic sequences include an n^2. It has a common second difference.

Fibonacci sequences are where you add the two previous terms to find the next term.

Linear/arithmetic sequence:

-3   +3   +3   +3   +3

+1   4   7   10   13   16.....

a) State the nth term

$3n + 1$

Difference   The 0th term

b) What is the 100th term in the sequence?

$3n + 1$

$3 \times 100 + 1 = 301$

c) Is 100 in this sequence?

$3n + 1 = 100$

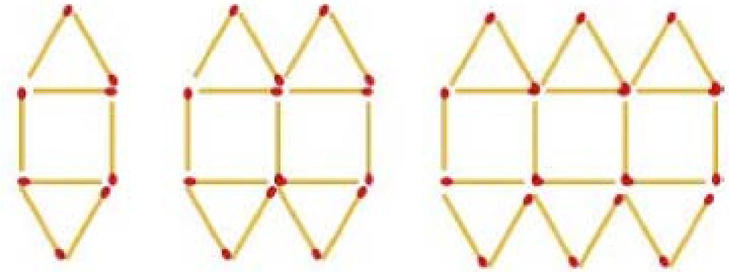
$3n = 99$

$n = 33$

Yes as 33 is an integer.

Examples

Pattern 1   Pattern 2   Pattern 3



Linear sequences with a picture:

State the nth term.

Hint: Firstly write down the number of matchsticks in each image:

$7n + 1$

Pattern 1	Pattern 2	Pattern 3
8	15	22

+1   -7   +7   +7

Geometric sequence e.g. ×3 ×3 ×3

4   12   36   108...

Quadratic sequence e.g. $n^2 + 4$ Find the first 3 numbers in the sequence

First term: $1^2 + 4 = 5$

Second term: $2^2 + 4 = 8$

Third term: $3^2 + 4 = 13$

Y9 Foundation

Key Words

Linear
Arithmetic
Geometric
Sequence
Nth term

1) 1, 8, 15, 22, ...

a) Find the nth term   b) Calculate the 50th term   c) Is 120 in the sequence?

2) $n^2 - 5$ Find the first 4 terms in this sequence

ANSWERS: 1a) $7n - 6$ b) 344 c) 18 so yes as n is an integer 2) -4, -1, 4, 11

Maths Knowledge Organiser

SEQUENCES

Key Concepts

Arithmetic sequences increase or decrease by a common amount each time.

Quadratic sequences have a common 2nd difference.

Fibonacci sequences Add the two previous terms to get the next term

Geometric series has a common multiple between each term

Y9 Higher

Examples

Linear sequences:

4 , 7, 10, 13, 16.....

a) State the nth term

$3n + 1$

Difference

The 0th term

b) What is the 100th term in the sequence?

$3n + 1$

$3 \times 100 + 1 = 301$

c) Is 100 in this sequence?

$3n + 1 = 100$

$3n = 99$

$n = 33$

Yes as 33 is an integer.

Quadratic sequences:

$a + b + c$	3	9	19	33	51	
$3a + b$	6	10	14	18		First difference
$2a$	4	4	4			Second difference

$2a = 4$
$a = 2$

$3a + b = 6$
$3 \times 2 + b = 6$
$b = 0$

$a + b + c = 3$
$2 + 0 + c = 3$
$c = 1$

$2n^2 + 0n + 1 \rightarrow 2n^2 + 1$

Key Words

Linear
Quadratic
Arithmetic
Geometric
Sequence
Nth term

A) 1, 8, 15, 22, ...

1) Find the nth term b) Calculate the 50th term c) Is 120 in the sequence?

B) Find the nth term for:

1) 5, 12, 23, 38, 57, ... 2) 3, 11, 25, 45, 71, ...

ANSWERS: A1) $7n - 6$ 2) 344 3) 18 so yes as n is an integer B1) $2n^2 + n + 2$ 2) $3n^2 - n + 1$

Maths Knowledge Organiser

DISTANCE-TIME GRAPHS

Key Concepts

A **distance-time** graph, plots time against the distance away from a starting point.

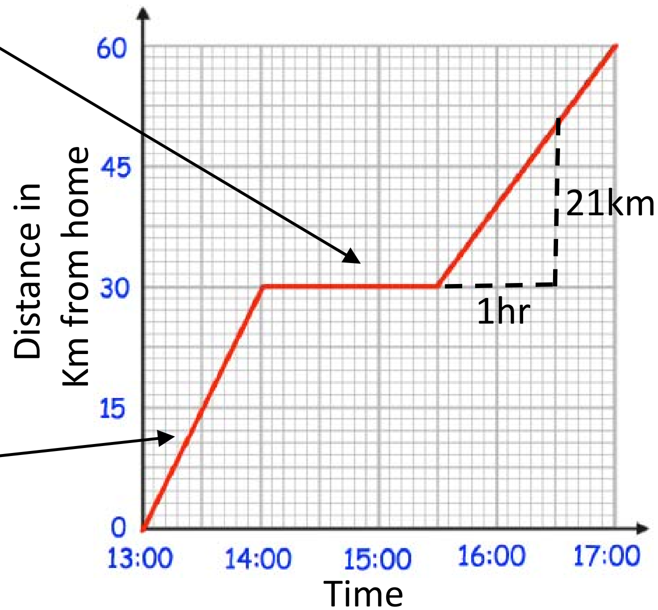
Speed can be calculated from these graphs by finding the gradient of the graph.

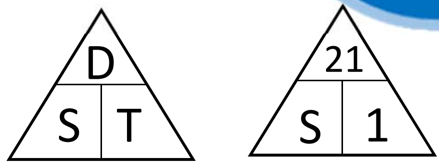
Horizontal lines are sections where the object is stationary.

Examples

Horizontal sections are where the object is stationary

Diagonal lines show the object moving away from home or moving closer to home





$$Speed = \frac{distance}{time}$$

$$Speed = \frac{21}{1}$$

$$Speed = 21km/h$$

Y9 Higher

Key Words

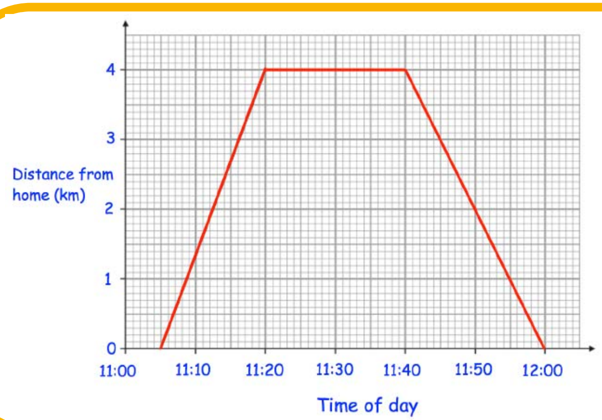
Distance

Time

Speed

Gradient

Stationary



A distance-time graph shows the journey of someone from home to the shop and back again.

1) How long were they at the shop for?
2) How far away from home is the shop?
3) How far did they travel in total?
4) What speed did they travel on the way to the shop in km/h?

ANSWERS: 1) 20 minutes 2) 4km 3) 8km 4) 16km/h

Maths Knowledge Organiser
STRAIGHT LINE GRAPHS AND EQUATION OF A

Key Concepts

Coordinates in 2D are written as follows:

x is the value that is to the left/right $\rightarrow (x, y) \leftarrow$ y is the value that is to up/down

Straight line graphs always have the equation:

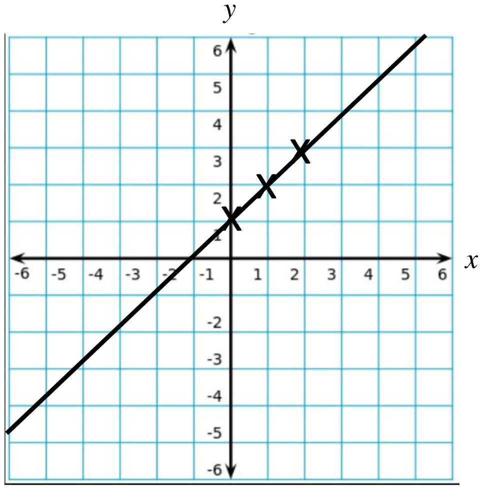
$$y = mx + c$$

m is the **gradient** i.e. the steepness of the graph.
c is the **y intercept** i.e. where the graph cuts the y axis.

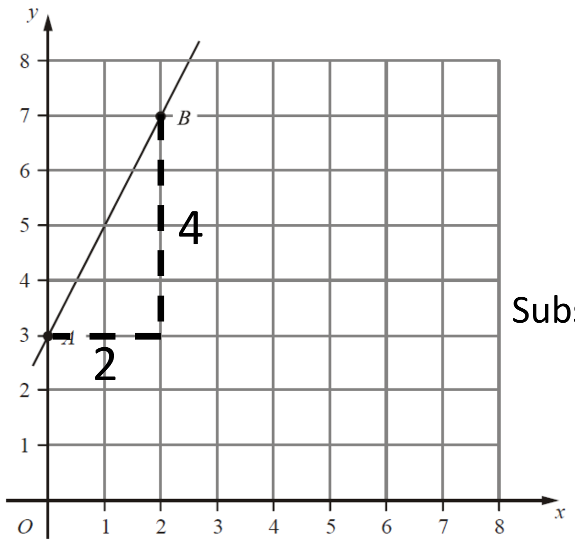
Examples

Plot the graph of $y = x + 1$

x	**0**	**1**	**2**
y	1	2	3



Calculate the equation of this line:



$$y = mx + c$$

$$m = \frac{4}{2}$$
$$= 2$$

$$y = 2x + c$$

Substitute in a coordinate: (2,7)

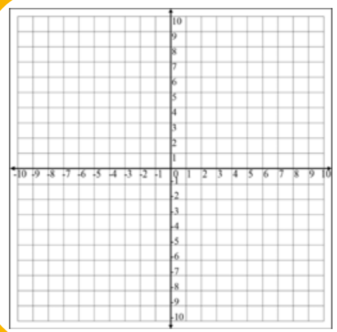
$$7 = (2 \times 2) + c$$

$$3 = c$$

$$y = 2x + 3$$

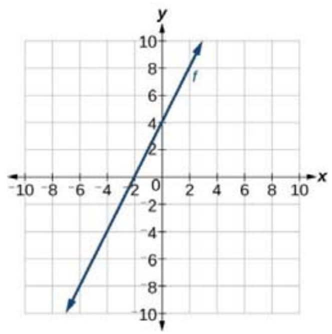
Y9 Higher

Key Words
Coordinate
Gradient



1) Plot the line $y = 3x - 2$

2) Find the equation of the line for the attached graph.



ANSWERS: 2) $y = 2x + 4$

Maths Knowledge Organiser

SOLVE SIMULTANEOUS EQUATIONS GRAPHICA

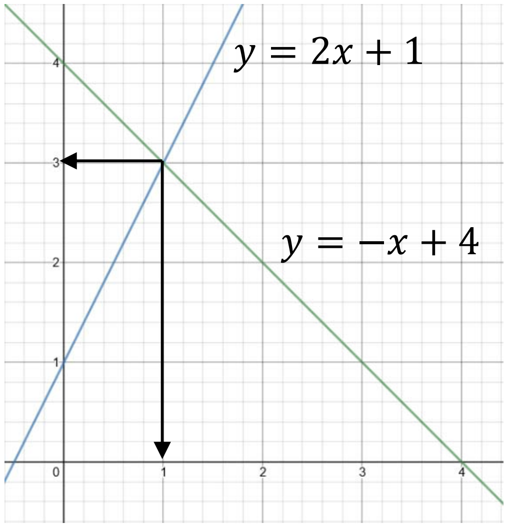
Key Concepts

Simultaneous equations are when **more than one equation** are given which involve **more than one variable**. The variables have the **same value** in each equation.

Simultaneous equations can be solved **graphically** whereby the **intersection** of the graphs gives the x and y values.

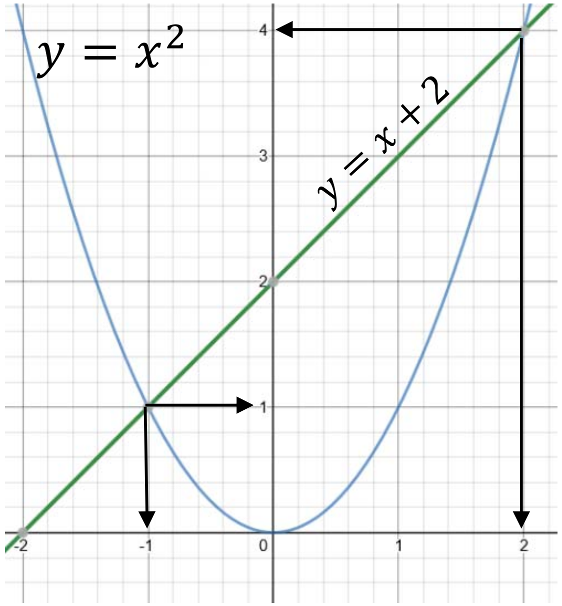
Examples

Solve graphically: $y = 2x + 1$
$y = -x + 4$



$x = 1$ and $y = 3$

Solve graphically: $y = x^2$
$y = x + 2$



$x = -1$ and $y = 1$

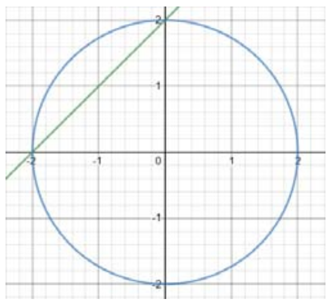
$x = 2$ and $y = 4$

Y9 Higher

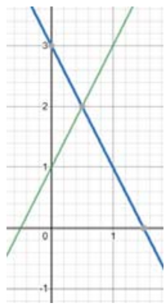
Key Words

Simultaneous
Equation
Intersection

1)



2)



Solve each set of simultaneous equations graphically.

ANSWERS 1) $x = -2$ and $y = 0$, $x = 0$ and $y = 2$ 2) $x = 0.5$ and $y = 2$

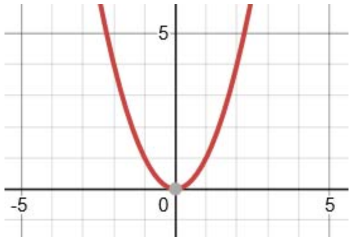
Maths Knowledge Organiser

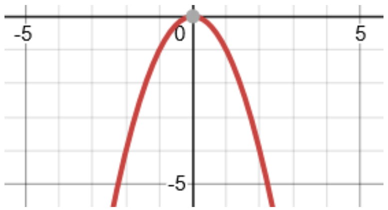
QUADRATIC GRAPHS

Key Concepts

A quadratic graph will always be in the shape of a parabola.

$y = x^2$ $\qquad$ $y = -x^2$

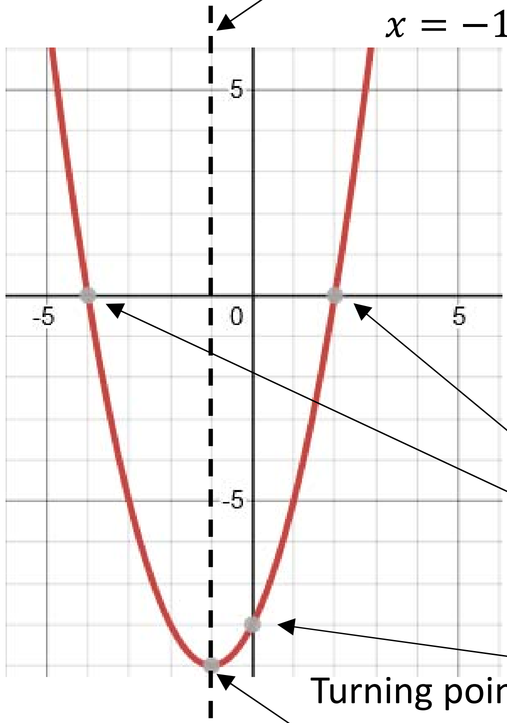




The roots of a quadratic graph are where the graph crosses the x axis. The roots are the solutions to the equation.

Examples

Line of symmetry
$x = -1$



Turning point
$(-1, -9)$

$$y = x^2 + 2x - 8$$

A quadratic equation can be solved from its graph.
The roots of the graph tell us the possible solutions for the equation.
There can be 1 root, 2 roots or no roots for a quadratic equation. This is dependant on how many times the graph crosses the x axis.

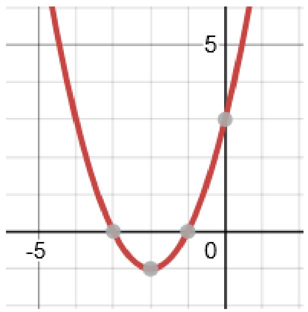
Roots $x = -4$
$x = 2$

y intercept = -8

Y9 Higher

Key Words

Quadratic
Roots
Intercept
Turning point
Line of symmetry



Identify from the graph of $y = x^2 + 4x + 3$:

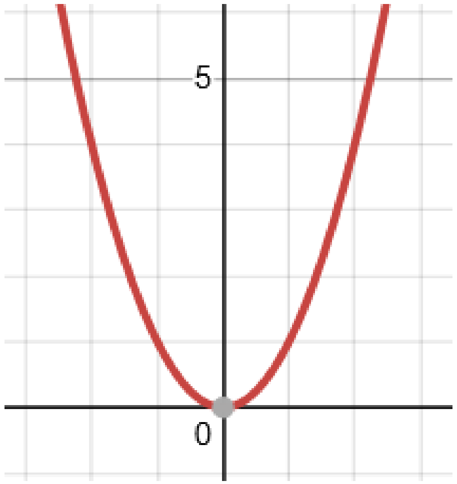
1) The line of symmetry
2) The turning point
3) The y intercept
4) The two roots of the equation

ANSWERS 1) $x = -2$ $\quad$ 2) $(-2, -1)$ $\quad$ 3) 3 $\quad$ 4) $x = -1$ *and* $x = -3$

Maths Knowledge Organiser

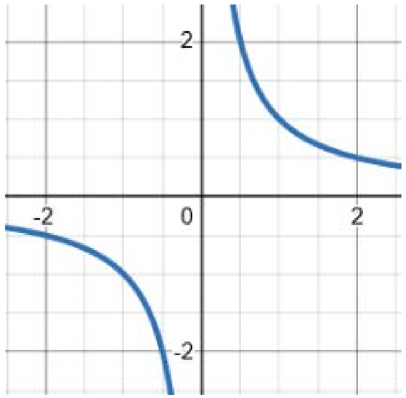
TYPES OF GRAPH

Examples



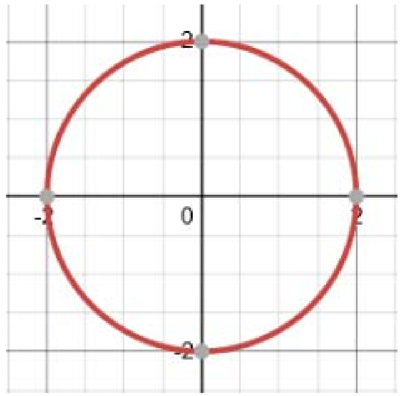
Quadratic graphs

$y = x^2$



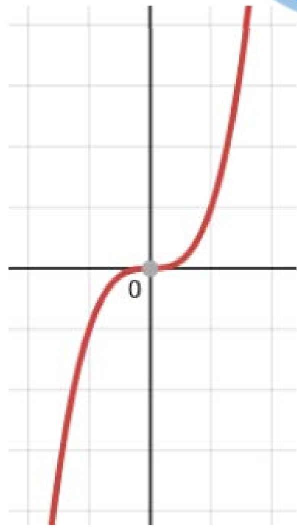
Reciprocal graphs

$y = \frac{1}{x}$



Circle graphs

$x^2 + y^2 = 4$



Cubic graphs

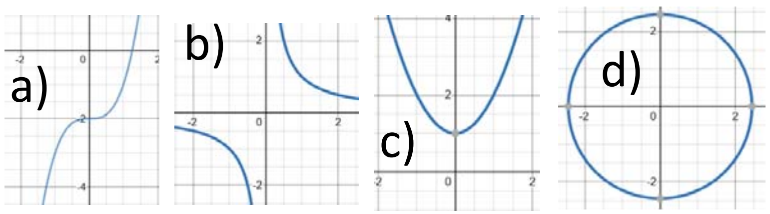
$y = x^3$

Y9 Higher

Key Words

Quadratic
Cubic
Reciprocal
Circle
Graph

Match the graph with the correct equation:



1) $x^2 + y^2 = 6$
2) $y = \frac{1}{x}$
3) $y = x^3 - 2$
4) $y = x^2 + 1$

ANSWERS: 1d 2b 3a 4c

Maths Knowledge Organiser

AREA AND PERIMETER OF BASIC SHAPES

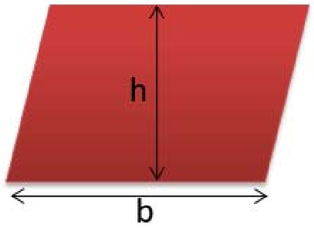
Key Concepts

The **area** of a 2D shape is the space inside it. It is measured in units squared e.g. cm^2

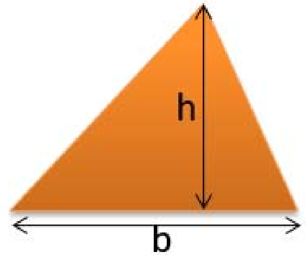
The **perimeter** of a shape is the distance around the edge of the shape. Units of length are used to measure perimeter e.g. mm, cm, m

A **compound shape** is a shape made up of others joined together.

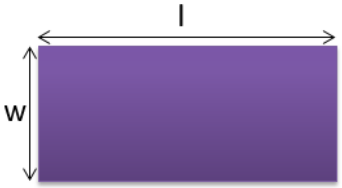
Examples



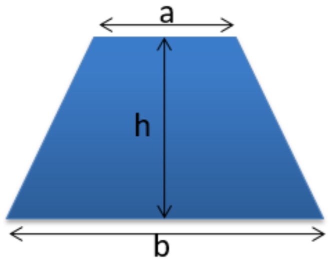
A = b × h



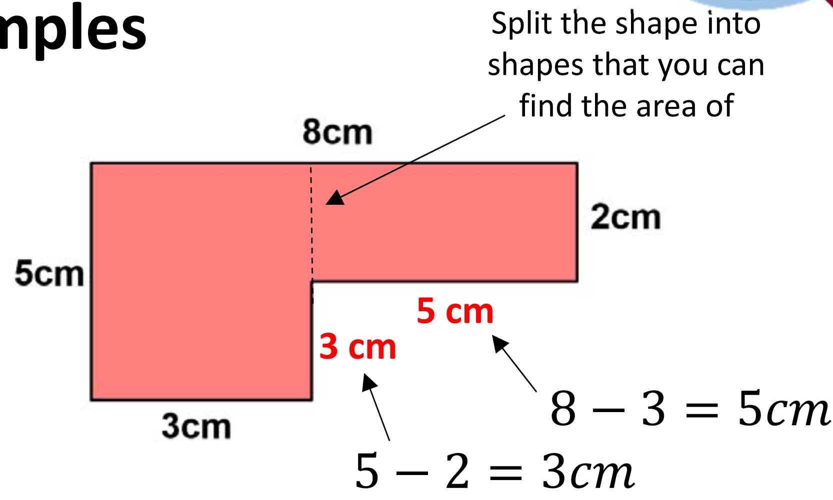
$$A = \frac{b \times h}{2}$$



A = l × w



$$A = \frac{(a + b) \times h}{2}$$



$$Area = (5 \times 3) + (2 \times 5)$$
$$= 25cm^2$$

$$Perimeter = 3 + 5 + 8 + 2 + 5 + 3$$
$$= 26cm$$

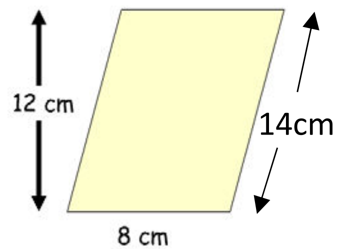
Y9 Higher

Key words

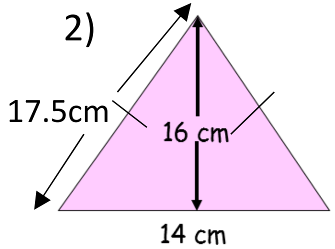
Area
Perimeter
Base
Height
Width
Length

Calculate the area and perimeter of each shape:

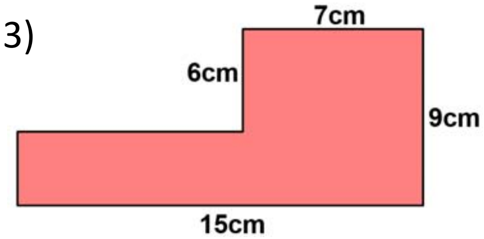
1)



2)



3)



ANSWERS: 1) A = 96 cm^2 P = 44cm 2) A = 112 cm^2 P = 49cm 3) A = 87 cm^2 P = 48cm

Maths Knowledge Organiser

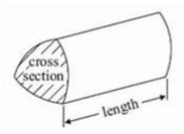
VOLUME AND SURFACE AREA OF PRISMS

Key Concept

The **volume** of an object is the amount of space that it occupies. It is measured in units cubed e.g. cm^3.

To calculate the volume of any prism we use:

$$\begin{matrix} \textit{area of} \\ \textit{cross section} \end{matrix} \times \textit{length}$$



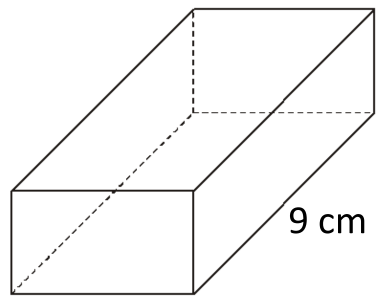
A **prism** is a 3D shape which has a continuous cross-section.

The **surface area** of an object is the sum of the area of all of its faces. It is measured in units squared e.g. cm^2.

Examples

$$\textbf{Volume} = 4 \times 9 \times 2$$

$$= \mathbf{72cm^3}$$



$\textbf{Surface area}$:

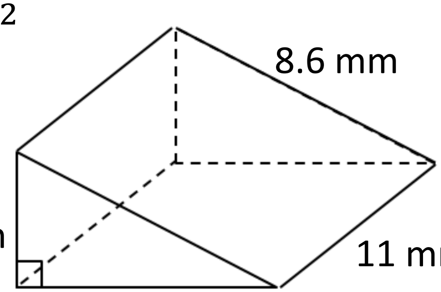
$$\begin{aligned} Front &= 4 \times 2 = 8 \\ Back &= 4 \times 2 = 8 \\ Side\ 1 &= 9 \times 2 = 18 \\ Side\ 2 &= 9 \times 2 = 18 \\ Bottom &= 4 \times 9 = 36 \\ Top &= 4 \times 9 = 36 \\ Total &= \mathbf{124cm^2} \end{aligned}$$

$$Area\ of\ triangle = \frac{5 \times 7}{2}$$

$$= 17.5mm^2$$

$$\textbf{Volume} = 17.5 \times 11$$

$$= \mathbf{192.5mm^3}$$



$\textbf{Surface area}$:

$$\begin{aligned} Front &= \frac{7 \times 5}{2} = 17.5 \\ Back &= \frac{7 \times 5}{2} = 17.5 \\ Side &= 5 \times 11 = 55 \\ Bottom &= 7 \times 11 = 77 \\ Top &= 11 \times 8.6 = 94.6 \\ Total &= \mathbf{261.6cm^2} \end{aligned}$$

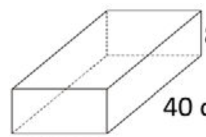
Y9 Higher

Key Words

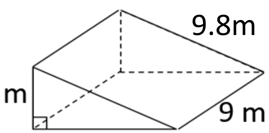
Volume
Capacity
Prism
Surface area
Face

Find the volume and surface area of each of these prisms:

1)



2)

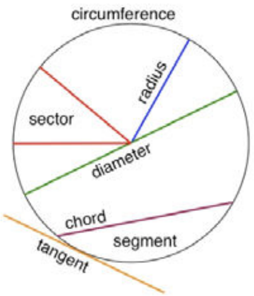


ANSWERS: 1)Volume = 5760 cm^3 Surface area = 2368 cm^2 2) Volume = 162 m^3 Surface area = 241.2m^2

Maths Knowledge Organiser
PERIMETER AND CIRCUMFERENCE

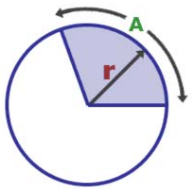
Key Concepts

Parts of a circle



Circumference of a circle is calculated by πd and is the distance around the circle.

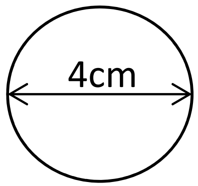
Arc length of a sector is calculated by $\frac{\theta}{360}\pi d$.



Examples

Calculate:

a) **Circumference**



C = $\pi \times 4$

= 4π

or = 12.57cm

b) **Diameter** when the circumference is 20cm

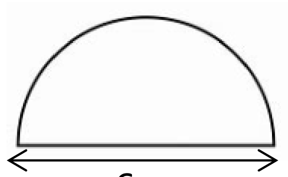
C = $\pi \times d$

20 = $\pi \times d$

$\frac{20}{\pi} = d$

Or $6.37cm$

c) **Perimeter**



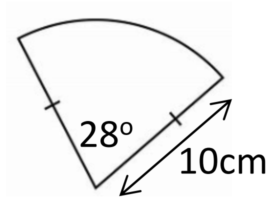
$$P = \frac{\pi \times d}{2} + d$$

$$P = \frac{\pi \times 6}{2} + 6$$

$$P = 3\pi + 6$$

Or = 15.42cm

d) **Arc length**



$$\text{Arc} = \frac{\theta}{360} \times \pi \times d$$

$$\text{Arc} = \frac{28}{360} \times \pi \times 2 \times 10$$

$$\text{Arc} = \frac{28}{360} \times \pi \times 20$$

$$\text{Arc} = \frac{14}{9}\pi$$

Or = 4.89cm

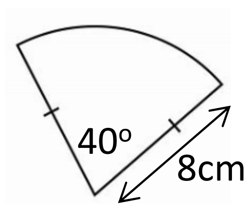
Y9 Higher

Key Words

Circle
Perimeter
Circumference
Radius
Diameter
Pi
Arc

Calculate:

1) The circumference of a circle with a diameter of 12cm
2) The diameter of a circle with a circumference of 30cm
3) The perimeter of a semicircle with diameter 15cm
4) The arc length of the diagram



ANSWERS: 1) 12π or 37.7cm 2) $\frac{30}{\pi}$ or 9.54cm 3) 38.56cm 4) $\frac{16}{9}\pi$ or 5.59cm

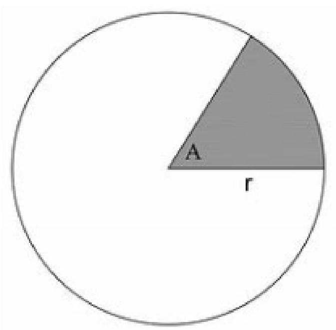
Maths Knowledge Organiser

AREA OF CIRCLES AND PART CIRCLES

Key Concepts

The **area** of a circle is calculated by πr^2

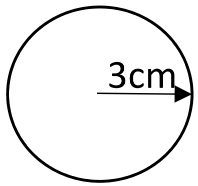
The **area of a sector** is calculated by $\frac{\theta}{360}\pi r^2$



Examples

Calculate:

a) **Area**



A = $\pi \times 3^2$

= 9π

or = 28.3cm^2

b) **Radius** when the area is 20cm^2

A = $\pi \times r^2$

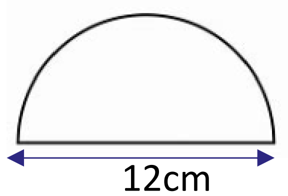
20 = $\pi \times r^2$

$\frac{20}{\pi} = r^2$

$\sqrt{\frac{20}{\pi}} = r$

Or $2.52cm$

c) **Area**



$P = \frac{\pi \times r^2}{2}$

$P = \frac{\pi \times 6^2}{2}$

$P = 18\pi$

Or = 56.55cm^2

d) **Area of a sector**

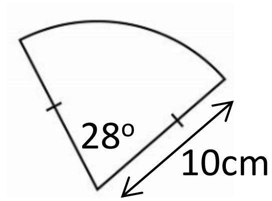
Arc $= \frac{\theta}{360} \times \pi \times r^2$

Arc $= \frac{28}{360} \times \pi \times 10^2$

Arc $= \frac{28}{360} \times \pi \times 100$

Arc $= \frac{70}{9}\pi$

Or = 24.43cm



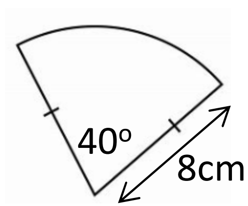
Y9 Higher

Key Words

Circle
Area
Radius
Diameter
Pi
Sector

Calculate:

1) The area of a circle with a radius of 9cm
2) The radius of a circle with an area of 45cm^2
3) The area of a semicircle with diameter of 16cm
4) The area of the sector in the diagram



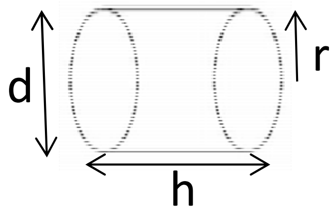
ANSWERS: 1) 81π or 254.47cm^2 2) $\sqrt{\frac{45}{\pi}}$ or 3.78cm^2 3) 32π or 100.53cm 4) $\frac{64}{9}\pi$ or 22.34cm

Maths Knowledge Organiser

VOLUME AND SURFACE AREAS OF CYLINDERS

Key Concepts

A **cylinder** is a **prism** with the cross section of a circle.

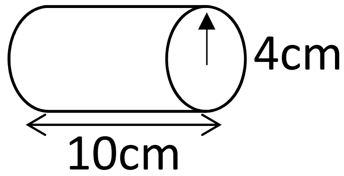


The **volume** of a cylinder is calculated by $\pi r^2 h$ and is the space inside the 3D shape

The **surface area** of a cylinder is calculated by $2\pi r^2 + \pi d h$ and is the total of the areas of all the faces on the shape.

Examples

From the diagram calculate:



a) **Volume**

$V = \pi \times r^2 \times h$

$V = \pi \times 4^2 \times 10$

$V = 160\pi$

Or $= 502.65cm^3$

b) **Surface Area –** You can use the net of the shape to help you

Area of two circles

$= 2 \times \pi \times r^2$

$= 2 \times \pi \times 4^2$

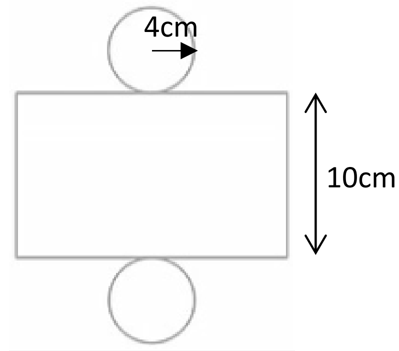
$= 32\pi$

Area of rectangle

$= \pi \times d \times h$

$= \pi \times 8 \times 10$

$= 80\pi$



$Surface\ Area = 32\pi + 80\pi$

$= 112\pi$

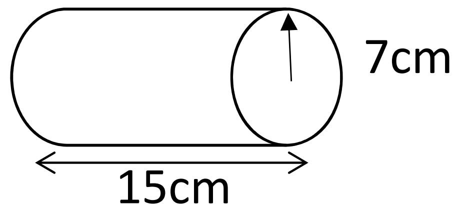
or $= 351.86cm^3$

Y9 Higher

Key Words

Cylinder
Surface Area
Radius
Diameter
Pi
Volume
Prism

Calculate the volume and surface area of this cylinder



ANSWERS: Volume = 735π or $2309.07cm^3$ Surface area = 308π or $967.61cm^3$

Maths Knowledge Organiser

BOUNDARIES

Key Concepts

The boundaries of a number derive from **rounding**.

E.g. State the boundaries of 360 when it has been rounded to 2 significant figures:

$$355 \leq x < 365$$

E.g. State the boundaries of 4.5 when it has been rounded to 2 decimal place:

$$4.45 \leq x < 4.55$$

These boundaries can also be called the **error interval** of a number.

Examples

	+	-	×	÷
Upper bound answer	$UB_1 + UB_2$	$UB_1 - LB_2$	$UB_1 \times UB_2$	$UB_1 \div LB_2$
Lower bound answer	$LB_1 + LB_2$	$LB_1 - UB_2$	$LB_1 \times LB_2$	$LB_1 \div UB_2$

When completing calculations involving boundaries we are aiming to find the greatest or smallest answer.

A restaurant provides a cuboid stick of butter to each table. The dimensions are 30mm by 30mm by 80mm, correct to the nearest 5mm. Calculate the upper and lower bounds of the volume of the butter.

$$Volume = l \times w \times h$$

$$Upper\ bound = 32.5 \times 82.5 \times 32.5 = 87140.63mm^3$$

$$Lower\ bound = 27.5 \times 77.5 \times 27.5 = 58609.38mm^3$$

$$D = \frac{x}{y}$$

x = 99.7 correct to 1 decimal place.
y = 67 correct to 2 significant figures.
Work out an upper and lower bounds for D.

$$Upper\ bound\ D = \frac{99.75}{66.5} = 1.5$$

$$Lower\ bound\ D = \frac{99.65}{67.5} = 1.48$$

Y9 Higher

Key Words

Bound
Upper
Lower
Accuracy
Rounding

1) Jada has 100 litres of oil, correct to the nearest litre.
The oil is poured into tins of volume 1.5 litres, correct to one decimal place.
Calculate the upper and lower bounds for the number of tins that can be filled.

2) There are 110 identical marbles in a bag. A marble is taken and weighed as 15.6 g to the nearest tenth of a gram. Find the upper and lower bounds for the weight of all the marbles.

ANSWERS: 1) LB = 69.3 ≈ 69 UB = 64.2 ≈ 64 2) LB = 1710.5 g UB = 1721.5 g